Formulario di Analisi Matematica II
versione provvisoria

Serie di MacLaurin :

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{+\infty} \frac{x^n}{n!} \qquad [R = \infty\ ]$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \sum_{n=0}^{+\infty} x^n \qquad [R = 1\ ,\ x \in (-1,1)\ ]$$

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots + (-1)^n x^{2n} + \cdots = \sum_{n=0}^{+\infty} (-1)^n x^{2n} \qquad [R = 1\ ,\ x \in (-1,1)\ ]$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots = \sum_{n=1}^{+\infty} (-1)^{n-1}\frac{x^n}{n} \qquad [R = 1\ ,\ x \in (-1,1]\ ]$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad [R = 1\ ,\ x \in [-1,1]\ ]$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad [R = \infty\ ]$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad [R = \infty]$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{+\infty} \frac{x^{2n+1}}{(2n+1)!} \qquad [R = \infty\ ]$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots) = \sum_{n=0}^{+\infty} \frac{x^{2n}}{(2n)!} \qquad [R = \infty\ ]$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \cdots = \sum_{n=0}^{+\infty} \binom{\alpha}{n} x^n \qquad [R = 1\ ,\ x \in (-1,1)\ ]$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \frac{35x^4}{128} + \cdots = \sum_{n=0}^{+\infty} \binom{-\frac{1}{2}}{n} x^n = \sum_{n=0}^{+\infty} (-1)^n \frac{(2n-1)!!}{(2n)!!} x^n \qquad [R = 1\ ,\ x \in (-1,1)\ ]$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \cdots = \sum_{n=0}^{+\infty} \binom{\frac{1}{2}}{n} x^n = 1 + \sum_{n=1}^{+\infty} (-1)^{n+1} \frac{(2n-3)!!}{(2n)!!} x^n \qquad [R = 1\ ,\ x \in (-1,1)\ ]$$

Serie di Fourier :

$$f(x) = a_0 + \sum_{k=1}^{\infty} a_k \cos(kx) + b_k \sin(kx),$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx, \qquad a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)\,dx \quad k \geq 1, \qquad b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,dx \quad k \geq 1.$$

$$\int_{-\pi}^{\pi} |f(x) - P_n(x)|^2\,dx = \int_{-\pi}^{\pi} f^2(x)\,dx - 2\pi a_0^2 - \pi \sum_{k=1}^{n} (a_k^2 + b_k^2)$$

$$\int_{-\pi}^{\pi} f^2(x)\,dx = 2\pi a_0^2 + \pi \sum_{k=1}^{\infty} (a_k^2 + b_k^2) \qquad \text{(identità di Parseval)}$$

Trasformata di Laplace :

$$\mathcal{L}[\,1\,] = \frac{1}{s} \qquad s > 0$$

$$\mathcal{L}[\,e^{-at}\,] = \frac{1}{s+a} \qquad s > -a$$

$$\mathcal{L}[\,t\,] = \frac{1}{s^2} \qquad s > 0$$

$$\mathcal{L}[\,t^n\,] = \frac{n!}{s^{n+1}} \qquad s > 0$$

$$\mathcal{L}[\,\sin(wt)\,] = \frac{w}{s^2 + w^2} \qquad s > 0$$

$$\mathcal{L}[\,\cos(\omega t)\,] = \frac{\omega s}{s^2 + \omega^2} \qquad s > 0$$

$$\mathcal{L}[\,\sinh(at)\,] = \frac{a}{s^2 - a^2} \qquad s > |a|$$

$$\mathcal{L}[\,\cosh(at)\,] = \frac{s}{s^2 - a^2} \qquad s > |a|$$

$$\mathcal{L}[\,f(t-a)H(t-a)\,] = e^{-as}\mathcal{L}[\,f\,](s)$$

$$\mathcal{L}[\,e^{at}f(t)\,] = \mathcal{L}[\,f\,](s-a)$$

$$\mathcal{L}[\,f'\,] = s\mathcal{L}[\,f\,] - f(0)$$

$$\mathcal{L}[\,f''\,] = s^2\mathcal{L}[\,f\,] - sf(0) - f'(0)$$

$$\mathcal{L}\left[\int_0^t f(x)\,dx\right] = \frac{1}{s}\,\mathcal{L}[\,f\,]$$

$$\mathcal{L}[\,tf(t)\,] = -\frac{d}{ds}\left(\mathcal{L}[\,f\,]\right)$$

$$\mathcal{L}[\,t^n f(t)\,] = (-1)^n\,\frac{d^n}{ds^n}\left(\mathcal{L}[\,f\,]\right)$$

$$\mathcal{L}\left[\frac{f(t)}{t}\right] = \int_s^{+\infty} \mathcal{L}[\,f\,](x)\,dx$$