

Some remarks on Bell's Inequality tests

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Abstract

We emphasize the difficulties of an experiment that can definitely discriminate between local realistic hidden variables theories and quantum mechanics using the Bell CHSH inequalities and a real measurement apparatus. In particular we analyze some examples in which the noise in real instruments can alter the experimental results, and the nontrivial problem to find a real “fair sample” of particles to test the inequalities.

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1 Introduction

In a classical paper [1], Einstein, Podolsky, and Rosen presented an argument which led them to infer that quantum mechanics is not a complete theory. They believed that the quantum mechanical description of a physical system could be improved supplementing it by new variables that can make quantum mechanics a local, realistic, complete and hopefully even deterministic theory. Postulating the existence of hidden variables, Bell proved an inequality (discussed in the next section) that must be satisfied by the local realistic theories but that is not fulfilled by the statistical predictions of quantum mechanics [2]. This inequality was generalized by Clauser, Horne, Shimony, and Holt [3], giving room to decisive tests between Quantum Mechanics (*QM*) and Local Hidden Variable (*LHV*) theories.

After some contradictory results [4] [5], between 1980 and 1982, Aspect et al. [6] [7] verified that in some experiments the results were correctly predicted by quantum theory and that in some configurations Bell's inequality was actually violated. At our knowledge all experiments so far support quantum theory [8][9][10].

However these results have been the subject of several criticisms, pointing out that there are “experimental loopholes” [11] [12] in Aspect's like experiments, divided essentially into two main classes: “locality” and “detection efficiency” loopholes, which might turn out to be responsible for the result. For example Franson [13] published a paper showing that the timing constraint in one of these experiments was not adequate to confirm that locality was violated. In a test with pairs of entangled photons meeting polarizers, a good measurement apparatus will rule out any possibility of subluminal communication which might inform the photon or the detector on one side about which measurement will be performed on the other side. For that purpose it has been suggested that the orientation of the polarizers must be determined by a purely random physical process and the photons must be registered with accurate time tags separately at each detector before any

information can be communicated from the other side. Recently Weihs et al. [14] have finally claimed to have close the locality loophole.

Other authors insist that all experiments so far have detected only a small subset of all pairs created [15], while Rowe et al [16] have recently performed an experiment without this problem. Later we will briefly comment about this loophole.

Moreover the existence of some “selection effect” which influences the detection probabilities has been suspected, so that Aspect’s experiments would not actually test Bell’s inequality [17]. So far no experiment aiming at testing Bell’s inequality seems to be simultaneously free by all possible loopholes [18][19][20][21].

Recently, additional motivations to investigate quantum non-locality have arisen, based on the potential applications of the fascinating field of quantum information processing: all of quantum computation and communication is based on the assumption that quantum systems can be entangled and that the entanglement can be maintained over long times and distances [22].

In this paper, starting from an ideal apparatus description, we review and emphasize some potential critical points of a real measurement system, in an information theory fashion. In particular we propose some examples in which the noise plays a role in Bell’s inequality tests and we consider the selection effect due to the choice of the sample of N particle pairs, independently of the detection efficiency.

2 Bell’s Inequality

Let us consider an ensemble of correlated pairs of particles moving so that one enters apparatus I and the other the apparatus II . There are adjustable apparatus parameters a and b . Each apparatus performs a measurement, giving in output the signals $A(a)$ and $B(b)$, respectively, where $A, B \in \{-, +\}$.

We distinguish between two cases.

In the LHV case we assume a statistical correlation between the mea-

surement results due to the information carried by and localized within each object. This information is part of the content of a set of hidden variables, denoted collectively by λ . So, owing to locality the joint probability on I and II is

$$P(A(a, \lambda), B(b, \lambda)) = P(A(a, \lambda))P(B(b, \lambda)) \quad (1)$$

mediated over λ .

In QM [23] “one can define *entangled* quantum states of two particles in such a way that their global state is perfectly defined, whereas the states of the separate particles remain totally undefined”. In this case the joint probability cannot be factored.

We define the correlation as

$$E(a, b) = P_{+,+}(a, b) + P_{-,-}(a, b) - P_{+,-}(a, b) - P_{-,+}(a, b) \quad (2)$$

The aim of the ideal experiment is to test the so called Bell-CHSH Inequality[3], stating that the sum

$$S = |E(a, b) - E(a, d)| + |E(c, b) + E(c, d)| \quad (3)$$

is ≤ 2 in *LHV* case, while QM predicts a violation of this inequality till the value $S = 2\sqrt{2}$ is reached.

2.1 Noise makes QM difficult to support

Each real measurement apparatus may exhibit a wrong measurement result. We model this as the classical BSC (Binary Symmetrical Channel) [24]: the signal is the correct one with probability $1 - \varepsilon$ and it is altered by noise with probability ε , where $0 \leq \varepsilon \leq 1$.

Let $P(A)$ be the probability of A in an ideal apparatus. Given

$$P(A) + P(\overline{A}) = 1 \quad (4)$$

where the overline means the opposite result, the probability of A in a noisy apparatus is

$$P_\varepsilon(A) = P(A)(1 - \varepsilon) + P(\overline{A})\varepsilon = P(A)(1 - 2\varepsilon) + \varepsilon \quad (5)$$

As also evidenced by Aspect [25] by no means we can consider the noise on I and II to be equal and independent of the apparatus parameters. So, in the LHV case the joint probability on I and II is

$$\begin{aligned} P_\varepsilon(A(a, \lambda), B(b, \lambda)) &= (P(A(a, \lambda))(1 - 2\varepsilon_1) + \varepsilon_1)(P(B(b, \lambda))(1 - 2\varepsilon_2) + \varepsilon_2) = \\ &= P(A(a, \lambda))P(B(b, \lambda))(1 - 2\varepsilon_1)(1 - 2\varepsilon_2) + P(A(a, \lambda))(1 - 2\varepsilon_1)\varepsilon_2 \\ &\quad + P(B(b, \lambda))(1 - 2\varepsilon_2)\varepsilon_1 + \varepsilon_1\varepsilon_2 \end{aligned} \quad (6)$$

and

$$\begin{aligned} P_\varepsilon(A(a), B(b)) &= P(A(a), B(b))(1 - \varepsilon_1)(1 - \varepsilon_2) + P(\bar{A}(a), B(b))\varepsilon_1(1 - \varepsilon_2) + \\ &\quad + P(A(a), \bar{B}(b))(1 - \varepsilon_1)\varepsilon_2 + P(\bar{A}(a), \bar{B}(b))\varepsilon_1\varepsilon_2 \end{aligned} \quad (7)$$

in the QM case.

It is easy to verify that in both cases the resulting noisy correlation is related to the ideal one by

$$E_\varepsilon(a, b) = (1 - 2\varepsilon_1)(1 - 2\varepsilon_2)E(a, b) \quad (8)$$

In this way S becomes

$$\begin{aligned} S_\varepsilon &= |(1 - 2\varepsilon_1) [(1 - 2\varepsilon_2)E(a, b) - (1 - 2\varepsilon_4)E(a, d)]| + \\ &\quad + |(1 - 2\varepsilon_3) [(1 - 2\varepsilon_2)E(c, b) + (1 - 2\varepsilon_4)E(c, d)]| \end{aligned} \quad (9)$$

The noise parameters have a non trivial role. For example assuming all such parameters to be equal to ε , S_ε is certainly lower than 2 $\forall \varepsilon > 0.15$ in both LHV and QM cases.

2.2 Noise makes LHV difficult to support, too

In testing Bell's inequality we need to compare the results at the two ends I and II of the apparatus. Since the measurement apparatus is a whole, the changing of one parameter has effects on the whole system. While in most cases this leads to a (3)'s value reduction, different results can be shown, as we will see in the following Gedankenexperiment.

Let us consider an example originally given by Bell [26]. A saw cuts a coin down the middle, so that the head and the cross are separated. The two halves (Half Head (H) and Half Cross (C)) are sent to the I and II buckets, respectively (see **Figure 1**).

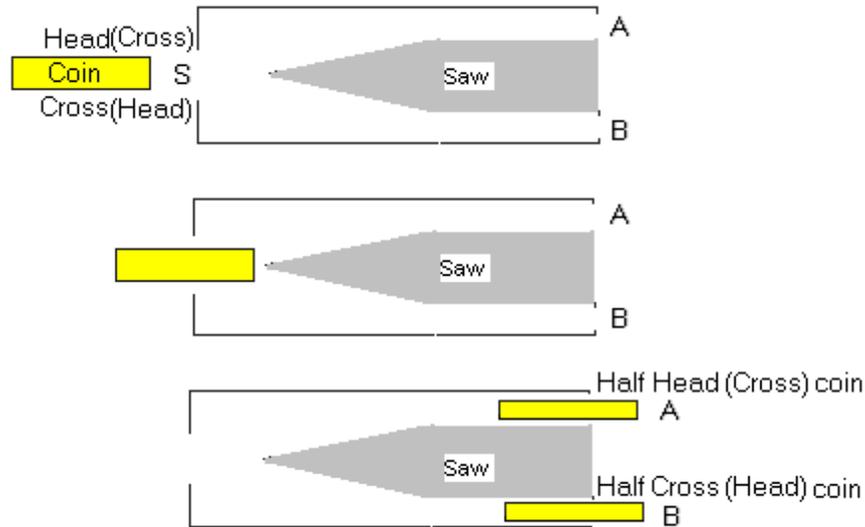


Figure 1: This figure shows a classical system in which a saw cuts a coin, so that the head and the cross are separated and sent into two different buckets. The upper part of the figure shows the apparatus and the full coin before entering the apparatus, the middle part shows the insertion of the coin, and the lower part the resulting two cut halves of the coin.

Over each bucket we place two cameras the first two to detect the head, the latter two to detect the cross. If there is detection, a signal is set to on in the circuit of **Figure 2**, otherwise it remains off. The circuit is made up of four stages L_1, L_2, L_3, L_4 each one including a counter of the concordance between each couple of incoming signals. Let the camera signals be represented as $A(a)$, $A(c)$, $B(b)$, and $B(d)$, each of which equals $+$ or $-$ according with the observation result, and where a, b, c , and d are adjustable parameters.

Each stage has two inputs. Each input signal is sent to a stage on a separate channel subject to noise. The particular conditions and the choice of a, b, c, d allow potentially a perfect recognition.

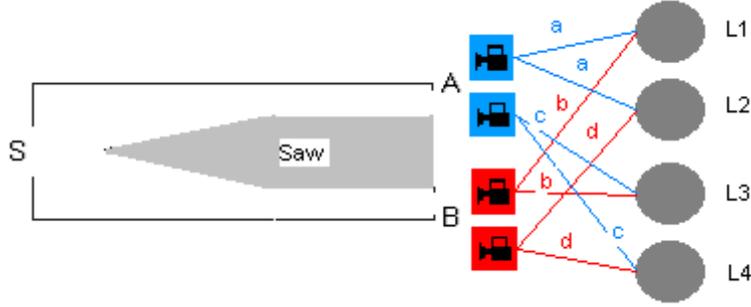


Figure 2: The figure shows the cutting-coin apparatus, the four cameras detecting the cross or the head on each half coin, and the circuit to count the concordance between each couple of incoming signals.

The probabilities of the camera signals are as reported in Table 1.

Table 1: Cameras.

A	B	$P(A(a), B(b))$	$P(A(a), B(d))$	$P(A(c), B(b))$	$P(A(c), B(d))$
+	+	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
+	-	0	0	0	0
-	+	0	0	0	0
-	-	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

Now let us assume the channel noises negligible except for $A(a)$ to L_2 . For example $A(a)$ may be not well connected to L_2 and the signal falls a certain percentage of times. This kind of error of misidentification is not so far from the reality of many experiments. For example Rowe et al.[16] admit that in their experiment a bright ion is misidentified 2% of the time as being dark.

Let us see the probabilities and correlation in the counter system, as reported in Table 2.

It is easy to verify that the Einstein-Bell locality has been preserved, but, by collecting the results in tables we have

$$|E_\varepsilon(a, b) - E_\varepsilon(a, d)| + |E_\varepsilon(c, b) + E_\varepsilon(c, d)| = 2 + \varepsilon \quad (10)$$

The result is that a macroscopic system can violate the Bell's inequality

Table 2: Counter system.

$L_i()$	$L_i()$	$P_\varepsilon(L_1(a), L_1(b))$	$P_\varepsilon(L_2(a), L_2(d))$	$P_\varepsilon(L_3(c), L_3(b))$	$P_\varepsilon(L_4(c), L_4(d))$
+	+	$\frac{1}{2}$	$\frac{1}{2}(1 - \varepsilon)$	$\frac{1}{2}$	$\frac{1}{2}$
+	-	0	0	0	0
-	+	0	$\frac{1}{2}\varepsilon$	0	0
-	-	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
		$E_\varepsilon(a, b) = 1$	$E_\varepsilon(a, d) = 1 - \varepsilon$	$E_\varepsilon(c, b) = 1$	$E_\varepsilon(c, d) = 1$

$\forall \varepsilon > 0$. In the light of this example the violation of Bell's inequality could be a good test to verify the existence of faulty instruments in a classical complex system.

2.3 Low Detection Efficiency may lead to unrepresentative results

Between the known loopholes in Bell's Inequality testing, the problem of the detection efficiency was identified as long ago as 1970, and it is also called the Enhancement Loophole. It arises because only a small subset of the pairs emitted are actually detected, so certain highly model-dependent assumptions have to be made about the statistical sampling[15][27].

We can model this fact by erasure channel [24]: the particle is detected with probability $1 - \delta$ and it is not detected by with probability δ , where $0 \leq \delta \leq 1$. So the probability of simultaneous detection is the product of the detection probability of each apparatus I and II , and only a fraction $(1 - \delta(A(a)))(1 - \delta(B(b)))$ of the correlated pairs is actually detected. When δ 's parameters grow we would have to assume that the sample of pairs registered is not a faithful representative of the whole ensemble emitted.

For example let us consider a sample of N particle pairs belonging to two distinct classes C_1 and C_2 such that $E_{C_1}(a, b) = 1$, $E_{C_2}(a, b) = -1$, and $\#C_1 = \#C_2$. Then $E(a, b) = 0$. Of the N couples we detect only a fraction ϕ . Then the probability of an equilibrate sampling between the two classes is just

$$\binom{N/2}{N\phi/2} \binom{N/2}{N\phi/2} \left(\frac{1}{2}\right)^{N\phi}$$

This corresponds to the introduction of free parameters Δ in S , leading to

$$S_\delta = |\Delta_1 E(a, b) - \Delta_2 E(a, d)| + |\Delta_3 E(c, b) + \Delta_4 E(c, d)| \quad (11)$$

where $-\frac{1}{E(a_i, b_i)} \leq \Delta_i \leq \frac{1}{E(a_i, b_i)}$

So we agree with Weihs et al. [14] that “an ultimate experiment should also have higher detection/collection efficiency, of the common 5% of actual experiments”, and that this loophole makes still possible “unlikely, local realistic or semiclassical interpretations”.

2.4 Ideal Detection Efficiency may lead to unrepresentative results, too

Even in the case of a perfect efficiency of the detectors (the Rowe’s experiment is the first to point in this direction), some problems might occur with the sample of N particles pairs used during the test.

Let us consider a Rowe et al. like experiment [16], involving four sets of phase angles chosen to apply the CHSH inequality. If we assume that hidden variables are in fact involved and the hidden variables of the N_1 pairs of the first set of phases are different from the N_2 of the second set, then the CHSH inequality becomes $S \leq 4$ [28] and we cannot discriminate between QM and LHV.

In order to avoid this problem, we can calculate the probability to have the values $\lambda_1, \lambda_2, \dots, \lambda_n$ of the first set N_1 equal to the $\lambda_1, \lambda_2, \dots, \lambda_n$ of N_2 . For example if $N_1 = N_2 = n$ and if λ can assume a discrete number of values N_{TOT} in the interval $0 \leq \lambda \leq 2\pi$ with an isotropic distribution (in the continuum case we would have $\rho(\lambda) = 1/2\pi$), we obtain

$$P = \prod_{i=0}^{n-1} \frac{n-i}{N_{TOT}-i} \quad (12)$$

In the case of Rowe et al. experiment $n = 20000$ but nobody knows N_{TOT} . If $N_{TOT} \rightarrow \infty$ we have $P \rightarrow 0$ and the experiment cannot discriminate between QM and LHV.

3 Conclusions

Bell's inequality tests necessitate major improvements of technology in order to finally, after more than 15 years, go significantly beyond the 1982 experiment of Aspect et al. [7]. While expecting that any improved experiment will also agree with quantum theory, actually the final answer to the eternal question: "Is the moon there, when nobody looks?", is certainly up to our judgement capability. But sometime also the question "Is the moon there when we look at it by a noisy telescope?" appears very hard to address.

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