Response to the Letter by Li and Cao*

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Abstract

Editor: In a previous work (Esposito, Rampone, & Tagliaferri, 1994), a neural net model which can be used to decode codewords belonging to a binary linear code and to correct noisy patterns which it receives from a transmission channel has been described. The net reduces the error probability to zero in the range of the error correcting capacity of the code, e, when applied to codewords whose maximum Hamming weight is not larger than $2e+3$. In a Letter to the Editor, Li and Cao (1997) proved theoretically that the network can be extended for all binary linear codes without limitations on maximum Hamming weight by means of a learning rule modification. On the other hand, in a previous article (Caianiello, Marinaro, Rampone, & Tagliaferri, 1996) we achieved the same result with a different network model, having the further advantages of binary connection weights and integer thresholds. We suggest that the two approaches be united by a further learning rule modification, as follows.

1 TRAINING PHASE

At each step Li and Cao (1997) update the coupling coefficients in the following way:

$$a'_{pk} = \begin{cases} \frac{1}{w_H(\underline{x}^h)} & \text{if } p = h,\ x^h_k = 1; \\ -\frac{1}{w_H(\underline{x}^h)} & \text{if } p = h,\ x^h_k = 0; \\ a_{pk} & \text{if } p \neq h \end{cases}$$

It is easy to see that each coupling coefficient a_{hk} of the A_h row is such that $|a_{hk}| = \frac{1}{w_H(\underline{x}^h)}$.

Therefore, by modifying the learning rule as

$$a'_{pk} = \begin{cases} 1 & \text{if } p = h,\ x^h_k = 1; \\ -1 & \text{if } p = h,\ x^h_k = 0; \\ a_{pk} & \text{if } p \neq h \end{cases}$$

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Li, L. and Cao, Z. (1997). A modified learning rule of the neural network for error correcting decoding. Neural Networks, 10, 387-388.

and by multiplying the threshold of neuron h by $w_H(\underline{x}^h)$, Theorem 1 in Esposito et al. (1994) and Theorem 2 in Li and Cao (1997) can still prove to be true, with the further advantage of binary connections weights.

2 THRESHOLD TUNING PHASE

In this phase, Li and Cao (1997) suggest decreasing the threshold θ_h of each neuron h by a quantity $e\Delta_h$,

$$\Delta_h = \frac{1}{w_H(\underline{x}^h)}$$

Using our new rule, we have just to multiply this quantity by $w_H(\underline{x}^h)$ to achieve the same error correcting properties, and in this way $\Delta_h = 1, \forall h$, so having Theorem 3 restated as follows:

Theorem 3 (further modified): Let $\underline{x}^h$ and $\underline{y}$ be the transmitted and the received vectors, respectively, with $d_H(\underline{y}, \underline{x}^h) \leq e$. Decreasing the threshold of each neuron by e, we have the firing of only one neuron, exactly the h one.

In this way, while retaining the network

1. reduces the error decoding probability to zero in the range of the erro correcting capacity of the code;

2. can be trained in a fast and simple way;

3. can be applied for all binary linear codes (Li and Cao modification);

we have the further advantages of binary weights and integer thresholds, except, obviously, for the ε factor, that can be neglected by substituting to the Heaviside function

$$1[x] = \begin{cases} 1 & \text{if } x > 0; \\ 0 & \text{if } x \leq 0; \end{cases}$$

the following activation function

$$\sigma[x] = \begin{cases} 1 & \text{if } x \geq 0; \\ 0 & \text{if } x < 0; \end{cases}$$

We should point out that the only requirement on a code to be used by this network, besides the minimum distance, is $w_{min}(C) \geq 2e + 1$, and not all the linear code properties.

However, when the codewords have not a natural structure, as for such codes, this requirement is rarely satisfied. In this sense, the 1996 model (Caianiello et al., 1996) remains more general, if a little more expensive.

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