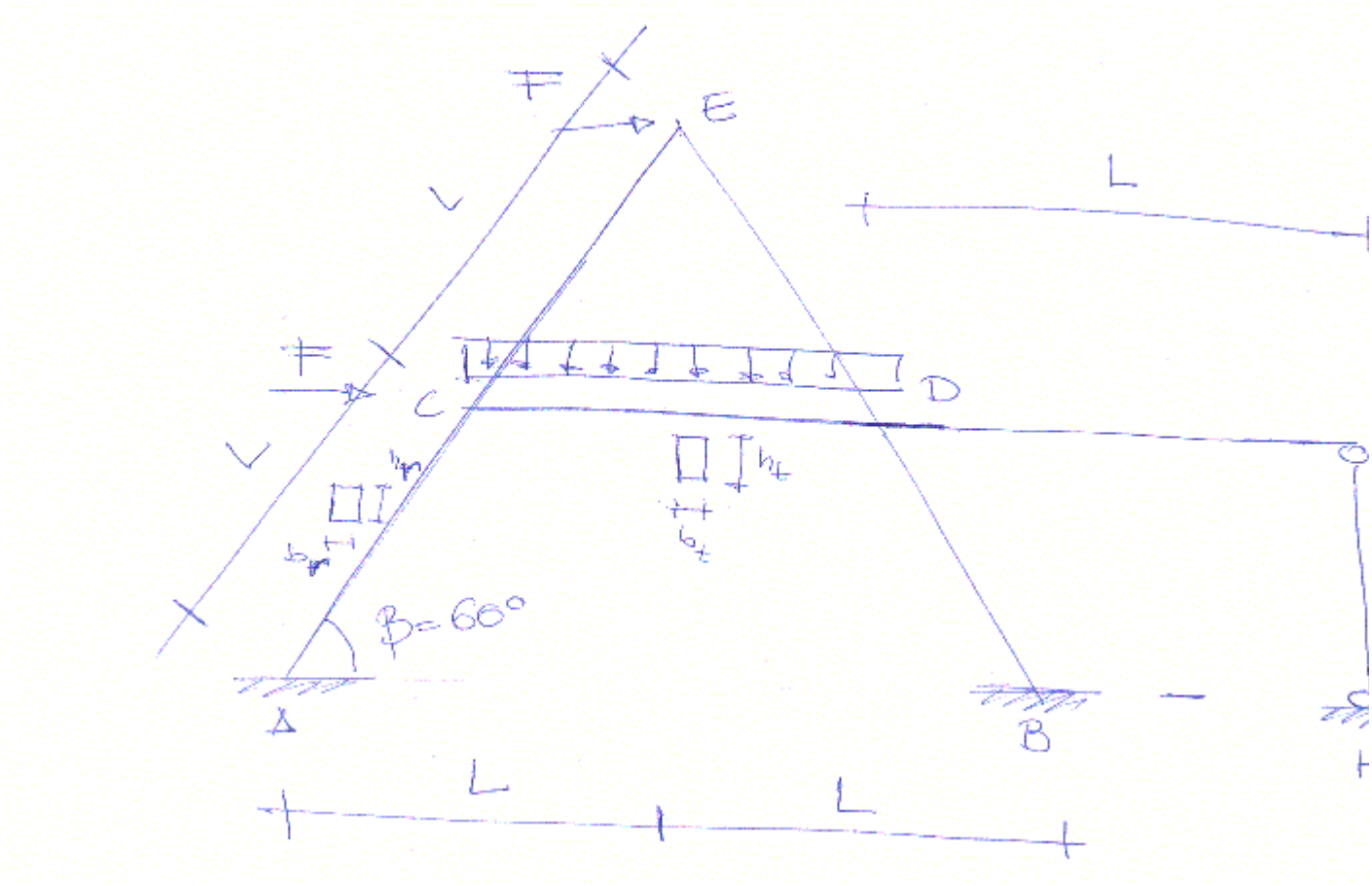
Schema Strutturale



Dati Numerici

In[406]:=
```
AngoloBeta = Pi / 3;
L = 5000;
bt = 300;
ht = 500;
br = 300;
hr = 400;
bp = 300;
hp = 300;
q = 40;
F = 20000;
fck = 20;
Lp = N[L/2 Tan[AngoloBeta]];
```

In[418]:=
```
It = bt ht^3 / 12;
Ir = br hr^3 / 12;
Ap = bp hp;
Ec = N[9500 (fck + 8)^(1 / 3)];
```

Incognite

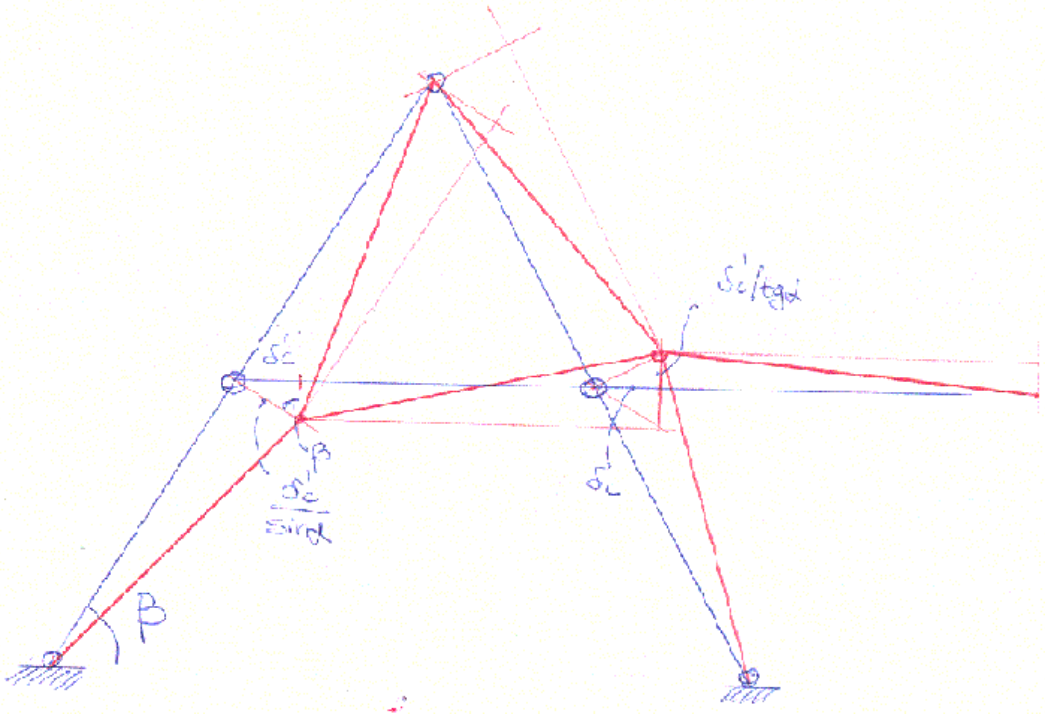
In[422]:=
```
Incognite = {FiC, FiD, FiE, DeltaC, DeltaG};
```

Espressione degli spostamenti d'asta in funzione di quelli dei nodi

Mobilitando uno alla volta i due spostamenti nodali che rappresentano i parametri da cui dipende la deformata cinematica (coordinate lagrangiane) del sistema si ottengono i due cinematismi rappresentati nel seguito:

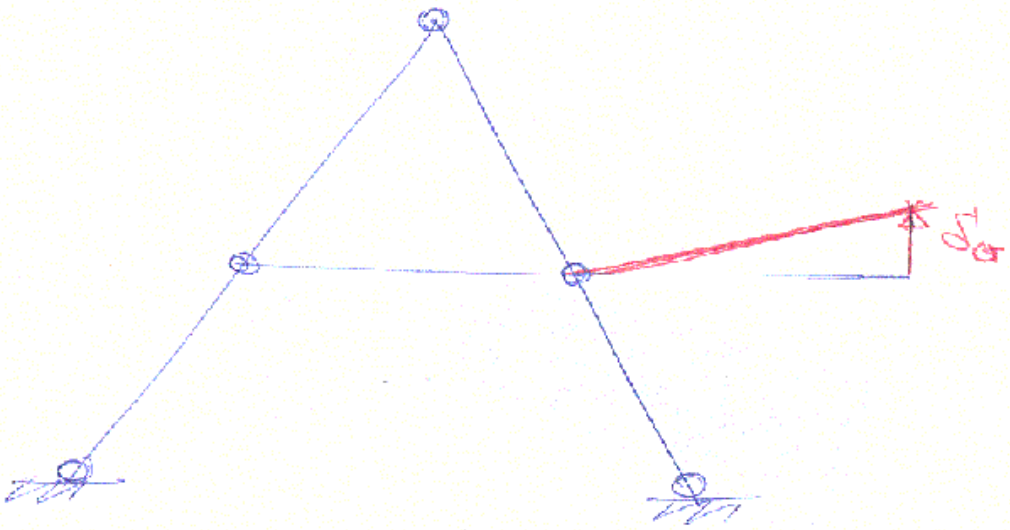
Cinematismo n.1

ASTE \ NODI	δ'_C
δ'_{AC}	$1/\sin\beta$
δ'_{BC}	$1/\sin\beta$
δ'_{CD}	$-2/\mathrm{tg}\,\beta$
δ'_{CE}	$-1/\sin\beta$
δ'_{DE}	$-1/\sin\beta$
δ'_{DG}	$1/\mathrm{tg}\,\beta$



Cinematismo n.2

ASTE \ NODI	δ'_G
δ'_{AC}	0
δ'_{BD}	0
δ'_{CD}	0
δ'_{CE}	0
δ'_{DE}	0
δ'_{DG}	-1



Sulla base di tali cinematismi è possibile risalire alle relazioni che legano gli spostamenti nodali (siano essi virtuali o effettivi) a quelli trasversali subiti dalle aste costruendo la seguente matrice di corrispondenza:

```
In[423]:= Tabella =
           {{1/Sin[AngoloBeta], 1/Sin[AngoloBeta],
              -2/Tan[AngoloBeta], -1/Sin[AngoloBeta],
              -1/Sin[AngoloBeta], 1/Tan[AngoloBeta]},
            {0, 0, 0, 0, 0, -1}};
          Tabella = Transpose[Tabella];
          MatrixForm[Tabella]
```

Out[425]//MatrixForm=

$$\begin{pmatrix} \frac{2}{\sqrt{3}} & 0 \\ \frac{2}{\sqrt{3}} & 0 \\ -\frac{2}{\sqrt{3}} & 0 \\ -\frac{2}{\sqrt{3}} & 0 \\ -\frac{2}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & -1 \end{pmatrix}$$

```
In[426]:= DeltaNodi = {DeltaC, DeltaG};
          DeltaVirtualeNodi =
           {DeltaVirtualeC, DeltaVirtualeG};
```

Definizione degli spostamenti trasversali delle aste

```
In[428]:= DeltaAste = {DeltaAC, DeltaBD, DeltaCD,
             DeltaCE, DeltaDE, DeltaDG}
          DeltaVirtualeAste =
           {DeltaVirtualeAC, DeltaVirtualeBD,
            DeltaVirtualeCD, DeltaVirtualeCE,
            DeltaVirtualeDE, DeltaVirtualeDG}

Out[428]= {DeltaAC, DeltaBD, DeltaCD,
           DeltaCE, DeltaDE, DeltaDG}

Out[429]= {DeltaVirtualeAC, DeltaVirtualeBD,
           DeltaVirtualeCD, DeltaVirtualeCE,
           DeltaVirtualeDE, DeltaVirtualeDG}
```

Legame tra spostamenti d'asta e spostamenti nodali

```
In[430]:= DeltaAste = Tabella.DeltaNodi
          DeltaVirtualeAste = Tabella.DeltaVirtualeNodi
```

$$\text{Out[430]= } \left\{\frac{2\,\text{DeltaC}}{\sqrt{3}}, \frac{2\,\text{DeltaC}}{\sqrt{3}}, -\frac{2\,\text{DeltaC}}{\sqrt{3}}, -\frac{2\,\text{DeltaC}}{\sqrt{3}}, -\frac{2\,\text{DeltaC}}{\sqrt{3}}, \frac{\text{DeltaC}}{\sqrt{3}} - \text{DeltaG}\right\}$$

Out[431]= $\left\{\frac{2\,\text{DeltaVirtualeC}}{\sqrt{3}},\ \frac{2\,\text{DeltaVirtualeC}}{\sqrt{3}},\ -\frac{2\,\text{DeltaVirtualeC}}{\sqrt{3}},\ -\frac{2\,\text{DeltaVirtualeC}}{\sqrt{3}},\ -\frac{2\,\text{DeltaVirtualeC}}{\sqrt{3}},\ \frac{\text{DeltaVirtualeC}}{\sqrt{3}},\ -\text{DeltaVirtualeG}\right\}$

Espressione dei coefficienti di rigidezza e dei momenti di incastro perfetto.

ASTA AC

In[432]:=
```
WAC = 4Ec Ir/L
WCA = 4Ec Ir/L
VAC = 2Ec Ir/L
VCA = 2Ec Ir/L
UAC = 6Ec Ir/L^2
UCA = 6Ec Ir/L^2
muCA = 0
muAC = 0
```

Out[432]= 3.69249×10^{10}

Out[433]= 3.69249×10^{10}

Out[434]= 1.84625×10^{10}

Out[435]= 1.84625×10^{10}

Out[436]= 1.10775×10^{7}

Out[437]= 1.10775×10^{7}

Out[438]= 0

Out[439]= 0

ASTA BD

In[440]:=
```
WBD = 4Ec Ir/L
WDB = 4Ec Ir/L
VBD = 2Ec Ir/L
VDB = 2Ec Ir/L
UBD = 6Ec Ir/L^2
UDB = 6Ec Ir/L^2
muBD = 0
muDB = 0
```

Out[440]= 3.69249×10^{10}

Out[441]= 3.69249×10^{10}

Out[442]= 1.84625×10^{10}

Out[443]= 1.84625×10^{10}

Out[444]= 1.10775×10^{7}

Out[445]= 1.10775×10^{7}

Out[446]= 0

Out[447]= 0

ASTA CD

```
In[448]:= WCD = 4 Ec It / L
          WDC = 4 Ec It / L
          VCD = 2 Ec It / L
          VDC = 2 Ec It / L
          UCD = 6 Ec It / L^2
          UDC = 6 Ec It / L^2
          muCD = -q L^2 / 12
          muDC = q L^2 / 12
```

Out[448]= 7.2119×10^{10}

Out[449]= 7.2119×10^{10}

Out[450]= 3.60595×10^{10}

Out[451]= 3.60595×10^{10}

Out[452]= 2.16357×10^{7}

Out[453]= 2.16357×10^{7}

Out[454]= $-\frac{250000000}{3}$

Out[455]= $\frac{250000000}{3}$

ASTA CE

```
In[456]:= WCE = 4 Ec Ir / L
          WEC = 4 Ec Ir / L
          VCE = 2 Ec Ir / L
          VEC = 2 Ec Ir / L
          UCE = 6 Ec Ir / L^2
          UEC = 6 Ec Ir / L^2
          muCE = 0
          muEC = 0
```

Out[456]= 3.69249×10^{10}

Out[457]= 3.69249×10^{10}

Out[458]= 1.84625×10^{10}

Out[459]= 1.84625×10^{10}

Out[460]= 1.10775×10^{7}

Out[461]= 1.10775×10^{7}

Out[462]= 0

Out[463]= 0

ASTA DE

```
In[464]:= WDE = 4 Ec Ir / L
          WED = 4 Ec Ir / L
          VDE = 2 Ec Ir / L
          VED = 2 Ec Ir / L
          UDE = 6 Ec Ir / L ^ 2
          UED = 6 Ec Ir / L ^ 2
          muDE = 0
          muED = 0
```

Out[464]= 3.69249×10^{10}

Out[465]= 3.69249×10^{10}

Out[466]= 1.84625×10^{10}

Out[467]= 1.84625×10^{10}

Out[468]= 1.10775×10^{7}

Out[469]= 1.10775×10^{7}

Out[470]= 0

Out[471]= 0

ASTA DG

```
In[472]:= WDG = 3 Ec It / L
          WGD = 0
          VDG = 0
          VGD = 0
          UDG = 3 Ec It / L ^ 2
          UGD = 0
          muDG = 0
          muGD = 0
```

Out[472]= 5.40892×10^{10}

Out[473]= 0

Out[474]= 0

Out[475]= 0

Out[476]= 1.08178×10^{7}

Out[477]= 0

Out[478]= 0

Out[479]= 0

Espressione dei momenti d'estremità delle varie aste

ASTA AC

In[480]:=
```
MAC = VAC FiC - UAC DeltaAste[[1]] + muAC
MCA = WCA FiC - UCA DeltaAste[[1]] + muCA
```

Out[480]= $-1.27912 \times 10^{7}\ \text{DeltaC} + 1.84625 \times 10^{10}\ \text{FiC}$

Out[481]= $-1.27912 \times 10^{7}\ \text{DeltaC} + 3.69249 \times 10^{10}\ \text{FiC}$

ASTA BD

In[482]:=
```
MBD = VBD FiD - UBD DeltaAste[[2]] + muBD
MDB = WDB FiD - UDB DeltaAste[[2]] + muDB
```

Out[482]= $-1.27912 \times 10^{7}\ \text{DeltaC} + 1.84625 \times 10^{10}\ \text{FiD}$

Out[483]= $-1.27912 \times 10^{7}\ \text{DeltaC} + 3.69249 \times 10^{10}\ \text{FiD}$

ASTA CD

In[484]:=
```
MCD = WCD FiC + VCD FiD - UCD DeltaAste[[3]] +
    muCD
MDC = WDC FiD + VDC FiC - UDC DeltaAste[[3]] + muDC
```

Out[484]= $-\frac{250000000}{3} + 2.49828 \times 10^{7}\ \text{DeltaC} + 7.2119 \times 10^{10}\ \text{FiC} + 3.60595 \times 10^{10}\ \text{FiD}$

Out[485]= $\frac{250000000}{3} + 2.49828 \times 10^{7}\ \text{DeltaC} + 3.60595 \times 10^{10}\ \text{FiC} + 7.2119 \times 10^{10}\ \text{FiD}$

ASTA CE

In[486]:=
```
MCE = WCE FiC + VCE FiE - UCE DeltaAste[[4]] + muCE
MEC = WEC FiE + VEC FiC - UEC DeltaAste[[4]] + muEC
```

Out[486]= $1.27912 \times 10^{7}\,\text{DeltaC} + 3.69249 \times 10^{10}\,\text{FiC} + 1.84625 \times 10^{10}\,\text{FiE}$

Out[487]= $1.27912 \times 10^{7}\,\text{DeltaC} + 1.84625 \times 10^{10}\,\text{FiC} + 3.69249 \times 10^{10}\,\text{FiE}$

ASTA DE

In[488]:=
```
MDE = WDE FiD + VDE FiE - UDE DeltaAste[[5]] + muDE
MED = WED FiE + VED FiD - UED DeltaAste[[5]] + muED
```

Out[488]= $1.27912 \times 10^{7}\,\text{DeltaC} + 3.69249 \times 10^{10}\,\text{FiD} + 1.84625 \times 10^{10}\,\text{FiE}$

Out[489]= $1.27912 \times 10^{7}\,\text{DeltaC} + 1.84625 \times 10^{10}\,\text{FiD} + 3.69249 \times 10^{10}\,\text{FiE}$

ASTA DG

In[490]:=
```
MDG = WDG FiD - UDG DeltaAste[[6]] + muDG
```

Out[490]= $-1.08178 \times 10^{7}\left(\frac{\text{DeltaC}}{\sqrt{3}} - \text{DeltaG}\right) + 5.40892 \times 10^{10}\,\text{FiD}$

Relazione tra reazione del pendolo e spostamenti incogniti

In[491]:=
```
Np = Ec Ap / Lp DeltaG
```

Out[491]= 599586. DeltaG

Scrittura delle equazioni di equilibrio

Equazioni di equilibrio alla rotazione

In[492]:=
```
Eq1 = MCA + MCD + MCE
Eq2 = MDB + MDC + MDE + MDG
Eq3 = MEC + MED
```

Out[492]= $-\frac{250000000}{3} + 2.49828 \times 10^{7}\,\text{DeltaC} + 1.45969 \times 10^{11}\,\text{FiC} + 3.60595 \times 10^{10}\,\text{FiD} + 1.84625 \times 10^{10}\,\text{FiE}$

Out[493]= $\frac{250000000}{3} + 2.49828 \times 10^{7}\,\text{DeltaC} - 1.08178 \times 10^{7}\left(\frac{\text{DeltaC}}{\sqrt{3}} - \text{DeltaG}\right) + 3.60595 \times 10^{10}\,\text{FiC} + 2.00058 \times 10^{11}\,\text{FiD} + 1.84625 \times 10^{10}\,\text{FiE}$

Out[494]= $2.55823 \times 10^{7}\,\text{DeltaC} + 1.84625 \times 10^{10}\,\text{FiC} + 1.84625 \times 10^{10}\,\text{FiD} + 7.38498 \times 10^{10}\,\text{FiE}$

Equazione di equilibrio globale

```
In[495]:= PLV = (MAC + MCA) DeltaVirtualeAste[[1]] / L +
            (MBD + MDB) DeltaVirtualeAste[[2]] / L +
            (MCD + MDC) DeltaVirtualeAste[[3]] / L +
            (MCE + MEC) DeltaVirtualeAste[[4]] / L +
            (MDE + MED) DeltaVirtualeAste[[5]] / L +
            MDG DeltaVirtualeAste[[6]] / L + F DeltaVirtualeC -
            Np DeltaVirtualeG;

In[496]:= Eq4 = -Coefficient[PLV, DeltaVirtualeC]
          Eq5 = -Coefficient[PLV, DeltaVirtualeG]
```

Out[496]= $-20000 + 35892.2\,\text{DeltaC} - 1249.14\,\text{DeltaG} + 2.49828 \times 10^{7}\,\text{FiC} + 1.87371 \times 10^{7}\,\text{FiD} + 2.55823 \times 10^{7}\,\text{FiE}$

Out[497]= $-1249.14\,\text{DeltaC} + 601750.\,\text{DeltaG} + 1.08178 \times 10^{7}\,\text{FiD}$

Costruzione della matrice di rigidezza e del vettore dei termini noti

```
In[498]:= Sistema = {Eq1, Eq2, Eq3, Eq4, Eq5};

In[499]:= MatriceK =
            Table[Table[Coefficient[Sistema[[i]], Incognite[[j]]],
              {j, 1, 5}], {i, 1, 5}];

In[500]:= MatrixForm[MatriceK]
```

Out[500]//MatrixForm=

$$\begin{pmatrix} 1.45969 \times 10^{11} & 3.60595 \times 10^{10} & 1.84625 \times 10^{10} & 2.49 \\ 3.60595 \times 10^{10} & 2.00058 \times 10^{11} & 1.84625 \times 10^{10} & 1.87 \\ 1.84625 \times 10^{10} & 1.84625 \times 10^{10} & 7.38498 \times 10^{10} & 2.55 \\ 2.49828 \times 10^{7} & 1.87371 \times 10^{7} & 2.55823 \times 10^{7} & 35 \\ 0 & 1.08178 \times 10^{7} & 0 & -12 \end{pmatrix}$$

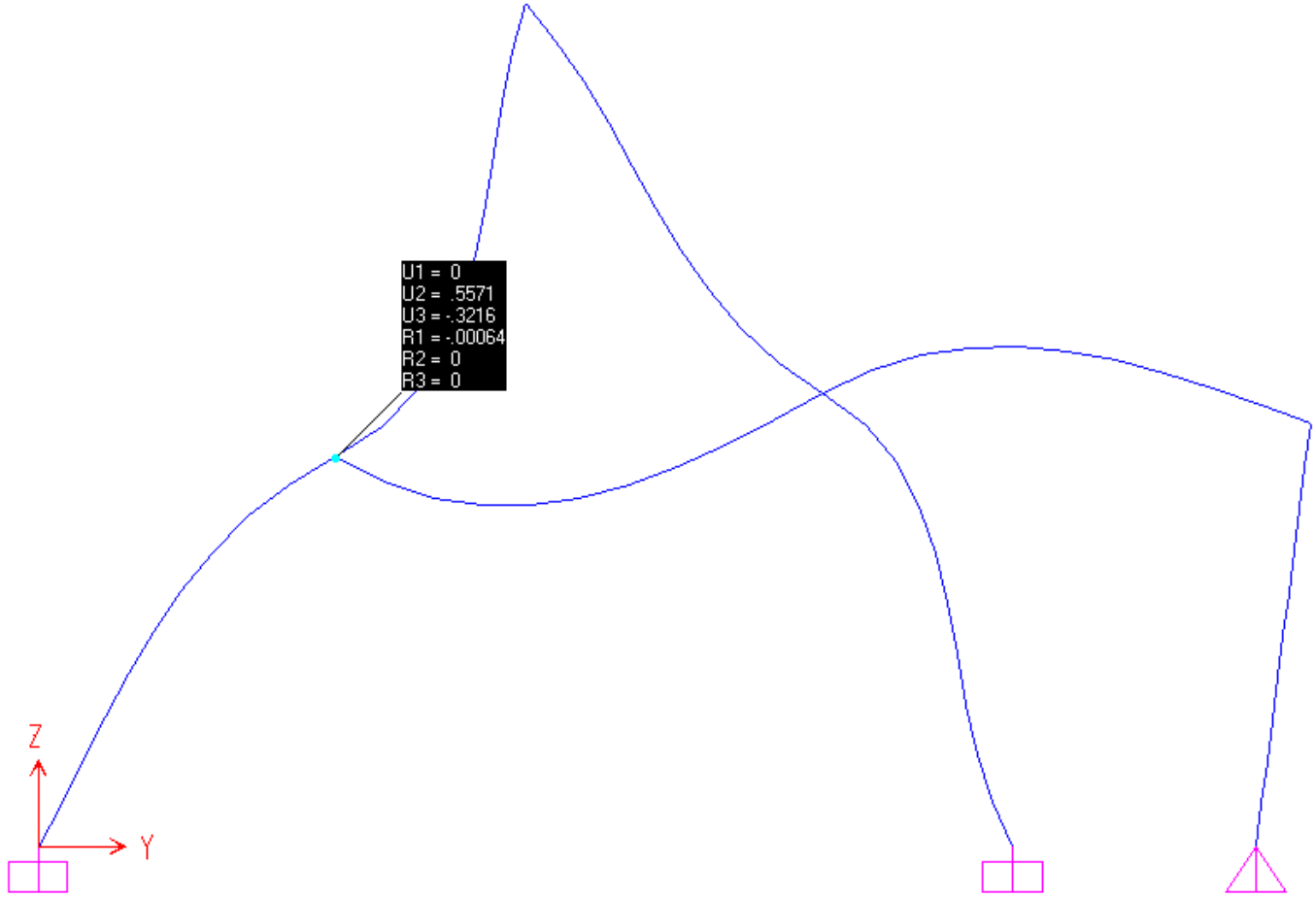
```
In[501]:= Q = -Sistema /. {FiC → 0, FiD → 0, FiE → 0,
              DeltaC → 0, DeltaG → 0}
```

Out[501]= $\left\{\frac{250000000}{3}, -\frac{250000000}{3}, 0, 20000, 0\right\}$

Soluzione delle equazioni di equilibrio

```
In[502]:= Sol = Solve[Sistema == 0, Incognite] // Flatten
```

Out[502]= {FiC → 0.000642076, FiD → -0.00056549, FiE → -0.000212135, DeltaC → 0.557109, DeltaG → 0.0113225}



Calcolo dei momenti nodali

ASTA AC

```
In[503]:= MACSol = MAC /. Sol
          MCASol = MCA /. Sol

Out[503]= 4.72823 × 10^6

Out[504]= 1.65825 × 10^7
```

ASTA BD

```
In[505]:= MBDSol = MBD /. Sol
          MDBSol = MDB /. Sol

Out[505]= -1.75664 × 10^7

Out[506]= -2.80067 × 10^7
```

ASTA CD

```
In[507]:= MCDSol = MCD /. Sol
          MDCSol = MDC /. Sol

Out[507]= -4.35007 × 10^7
```

Out[508]= 7.96218×10^7

ASTA CE

In[509]:= MCESol = MCE /. Sol
MECSol = MEC /. Sol

Out[509]= 2.69181×10^7

Out[510]= 1.11473×10^7

ASTA DE

In[511]:= MDESol = MDE /. Sol
MEDSol = MED /. Sol

Out[511]= -1.76711×10^7

Out[512]= -1.11473×10^7

ASTA DG

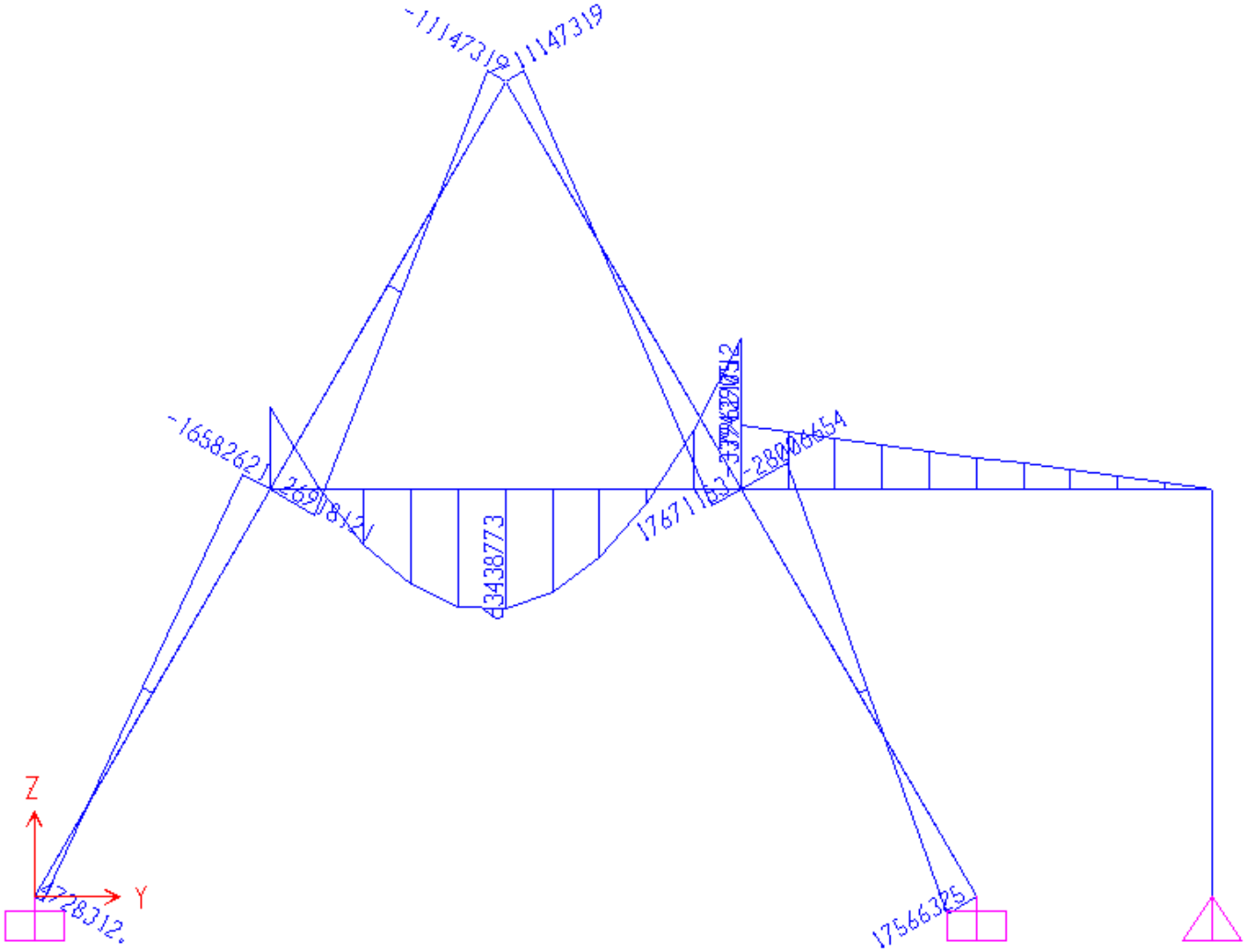
In[513]:= MDGSol = MDG /. Sol

Out[513]= -3.3944×10^7

Pendolo GH

In[514]:= NpSol = Ec Ap / Lp DeltaG /. Sol

Out[514]= 6788.79



Converted by *Mathematica* June 16, 2005